It’s magic! No, it’s algebra!!

Information for students

- The purpose of the problems in Appendix A is to deepen your knowledge of algebra.
- They will also strengthen your understanding of the formula to calculate the area of a trapezoid.
- Working through these problems may also help you develop a greater understanding of algebraic concepts and a greater appreciation for algebra.

Materials required

- Paper, writing materials (you should not use a calculator!)

Information for parents

- Start by encouraging your child to solve the problems without assistance (electronic or otherwise), but allow them to reach out to friends or teachers (or help them yourself) to find the answers through discussion.
- Use the hints related to each problem to provide that little extra push to help them along.

Appendix A

Problem 1

Here is a “magic” trick to impress your friends and family

#	Action	Result
1	Pick a number between 1 and 25.	
2	Add 9.	
3	Multiply the sum by 3.	
4	Subtract 6 from the product.	
5	Divide the result by 3.	
6	Subtract the original number from your final result.	
7	What is your answer?	

Repeat with a different number. What do you get? Write an algebraic expression that would allow you to find the solution for any number between 1 and 25.

Problem 2

The difference and product of these two numbers are equal. What characteristic displayed here makes this true for all similar types of equations?

$$\frac{3}{16} - \frac{3}{19} = \frac{3}{16} \times \frac{3}{19}$$

Use algebra to explain why this works. Start with this: $\frac{a}{b} - \frac{a}{c} = \frac{a}{b} \times \frac{a}{c}$

Problem 3

The formula for the area of a trapezoid is as follows:

$\boldsymbol{A = \frac{(B+b)h}{2}}$; where b is the length of the small base, B is the length of the large base and h is the height of the trapezoid. (see Figure 1)

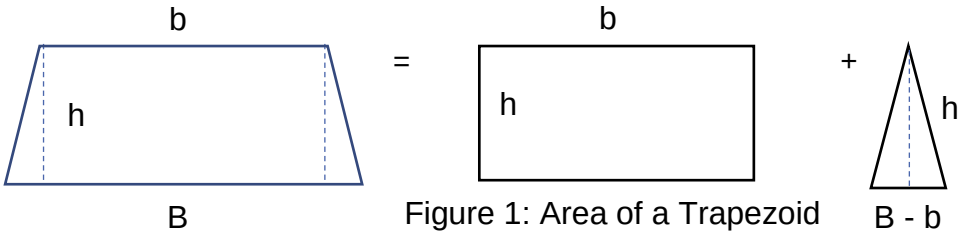


Figure 1: Area of a Trapezoid

Show that the formula for the total area of the trapezoid can be found by adding the area of the rectangle, b X h, to the total area of the triangles found at both ends of the trapezoid and given by $\boldsymbol{\frac{(B-b)h}{2}}$.

Use your knowledge of algebra to simplify the formula below and compare it to the standard formula provided above.

$$\boldsymbol{A = b \times h + \frac{(B-b)h}{2}}$$

Appendix B – Hints and Solutions

Problem 1

Hints to help students and encourage them to explore the topic:

- Tell your child/student to perform the operations with any number (i.e. 1).
- Encourage them to write down each step (i.e. 1 + 9, 3(1 + 9), etc.).
- Encourage them to create another series of operations that would be “magical.” What would that look like algebraically?
- Encourage them to develop another series of operations, but using more complex operations (i.e. square, square roots, exponents).

Solution

#	Action	Result
1	Pick a number between 1 and 25.	$\boldsymbol{x}$
2	Add 9.	$\boldsymbol{x + 9}$
3	Multiply the sum by 3.	$\boldsymbol{3(x + 9)}$
4	Subtract 6 from the product.	$\boldsymbol{3(x + 9) - 6}$
5	Divide the result by 3.	$\boldsymbol{\frac{3(x + 9) - 6}{3}}$
6	Subtract the original number from your final result.	$\boldsymbol{\frac{3(x + 9) - 6}{3} - x}$
7	What is your answer?	$\boldsymbol{7}$

Problem 2

Hints to help students and encourage them to explore the topic:

- Use the steps of the answer provided as hints to allow your child/student to move forward.
- Ask them how fractions are to be subtracted (they must have the same denominator).
- Remind them that whatever they do on one side of the equation, they must do on the other side of the equation.
- Ask them to create another equation with different fractions, using the relationship observed, to verify if the rule remains true.

Solution

$$\frac{a}{b} - \frac{a}{c} = \frac{a}{b} \times \frac{a}{c}$$

$$\frac{ac}{bc} - \frac{ab}{bc} = \frac{a^2}{bc}$$

$$ac - ab = a^2$$

$$a(c - b) = a^2$$

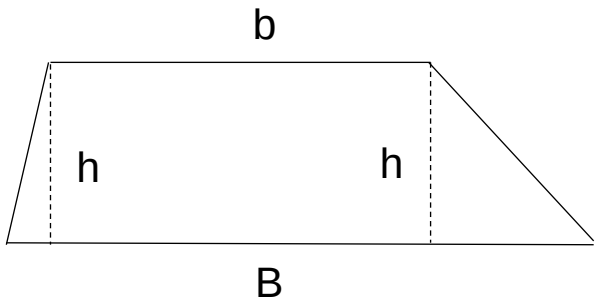
$$\frac{a(c - b)}{a} = \frac{a^2}{a}$$

$$c - b = a$$

Problem 3

Hints to help students and encourage them to explore the topic:

- Help children/students see the triangular portions of the trapezoid (the right triangles at the ends of the figure) and how they combine to form an isosceles triangle.
- Ask them if they understand why breaking the trapezoid down into a rectangle and a triangle (or two) works?
- Does the formula work for a trapezoid that is not an isosceles trapezoid? Using algebra and given the following figure, show (explain) why it is still true.



Solution

$$\boldsymbol{A = b \times h + \frac{(B - b)h}{2}}$$

$$\boldsymbol{A = \frac{2bh}{2} + \frac{Bh - bh}{2}}$$

$$\boldsymbol{A = \frac{2bh + Bh - bh}{2}}$$

$$\boldsymbol{A = \frac{bh + Bh}{2}}$$

$$\boldsymbol{A = \frac{h(b + B)}{2}}$$