Find the Mistake[1]

Information for students

This task will help you strengthen your understanding of the laws of exponents.

Instructions

- Examine the expressions provided in Appendix A – Find the Mistake.
- Identify the four (4) INCORRECT expressions.
- For each INCORRECT expression, identify the mistake and apply the reasoning needed to correct it.
- If the expression is CORRECT, explain (show) why it is correct.

Materials required

- Appendix A – Find the Mistake
- Writing materials
- Calculator

Information for parents

About the activity

Children should:

- complete the activity on their own
- refer to various sources to review the laws of exponents (class notes, textbooks, internet sources, etc.)

Parents could:

- help the children organize the required materials, if necessary
- read the instructions to the children, if necessary
- have the children explain how they went about determining which statements were correct and incorrect and ask them to explain why

[1] Task adapted from Andrew Stadel, "Thank You Math Mistakes," April 16, 2013, http://mr-stadel.blogspot.com/2013/04/thank-you-math-mistakes.html

Appendix A – Find the Mistake

Information for students

This task will help you strengthen your understanding of the laws of exponents.

Instructions

- Examine the expressions below.
- Identify the four (4) INCORRECT expressions.
- For each INCORRECT expression, identify the mistake and apply the reasoning needed to correct it.
- If the expression is CORRECT, explain (show) why it is correct.

1. $z^5 \cdot z \cdot z^7 = z^{12}$

2. $(4g)^{-3} = \frac{1}{64g^3}$

3. $\frac{(-4)^7}{(-4)^4} = 64$

4. $(5pq)^3 = 125pq$

5. $\frac{3^5}{3^9} = \frac{1}{81}$

6. $(-20x^3)^2(x^7) = 400x^{13}$

7. $\left(\frac{x^2}{3y^3}\right)^2 = \frac{x^4}{9y^6}$

8. $(4x^{-2}y^3)^{-3} = \frac{x^6y^9}{64}$

Appendix B – Solutions

Equation	Answer
$z^5 \cdot z \cdot z^7 = z^{12}$	INCORRECT The *Law of Products with the Same Base* states that when multiplying like bases, keep the base the same and add the exponents ($x^a \cdot x^b = x^{a+b}$). According to this law, the equality should be: $z^5 \cdot z \cdot z^7 = z^{5+1+7} = \boldsymbol{z^{13}}$
$(4g)^{-3} = \frac{1}{64g^3}$	CORRECT This equation is based on the *Law of Negative Exponents* as well as the *Law of Product to a Power*. The Law of Negative Exponents states that negative exponents signify division. In particular, find the reciprocal of the base ($x^{-a} = \frac{1}{x^a}$). The Law of Product to a Power states that when raising a product to a power, distribute the power to each factor ($(xy)^a = x^a y^a$). According to these laws, the expression on the left can be simplified as follows: $(4g)^{-3} = \frac{1}{(4g)^3} = \frac{1}{4^3 \cdot g^3} = \frac{1}{64g^3}$
$\frac{(-4)^7}{(-4)^4} = 64$	INCORRECT The *Law of Quotients with the Same Base* states that when dividing like bases, keep the base the same and subtract the exponent of the denominator from the exponent of the numerator ($\frac{x^a}{x^b} = x^{a-b}$). According to this law, the equality should be: $\frac{(-4)^7}{(-4)^4} = (-4)^3 = (-4)(-4)(-4) = -64$

$(5pq)^3 = 125pq$	INCORRECT The *Law of Product to a Power* states that when raising a product to a power, distribute the power to each factor ($(xy)^a = x^a y^a$). According to this law, the equality should be: $(5pq)^3 = 5^3 \cdot p^3 \cdot q^3 = 125p^3q^3$
$\frac{3^5}{3^9} = \frac{1}{81}$	CORRECT The *Law of Quotients with the Same Base* states that when dividing like bases, keep the base the same and subtract the exponent of the denominator from the exponent of the numerator ($\frac{x^a}{x^b} = x^{a-b}$). The *Law of Negative Exponents* states that negative exponents signify division. In particular, find the reciprocal of the base ($x^{-a} = \frac{1}{x^a}$). According to these laws, the expression on the left can be simplified as follows: $\frac{3^5}{3^9} = 3^{5-9} = 3^{-4} = \frac{1}{3^4} = \frac{1}{81}$
$(-20x^3)^2(x^7) = 400x^{13}$	CORRECT This equation is based on the *Law of Products with the Same Base* as well as the *Law of a Power to a Power*. The Law of Products with the Same Base states that when multiplying like bases, keep the base the same and add the exponents ($x^a \cdot x^b = x^{a+b}$). The Law of a Power to a Power states that when raising a base with a power to another power, keep the base the same and multiply the exponents ($(x^b)^a = x^{ab}$). According these laws, the expression on the left can be simplified as follows: $(-20x^3)^2(x^7) = (-20)^2(x^3)^2(x^7) = 400x^{2\cdot 3}x^7 = 400x^6x^7 = 400x^{6+7} = 400x^{13}$
$\left(\frac{x^2}{3y^3}\right)^2 = \frac{x^4}{9y^6}$	CORRECT This equation is based on the *Law of a Power to a Power*. The Law of a Power to a Power states that when raising a base with a power to another power, keep the base the same and multiply the exponents ($(x^b)^a = x^{ab}$). According to this law, the expression on the left can be simplified as follows: $\left(\frac{x^2}{3y^3}\right)^2 = \frac{(x^2)^2}{(3)^2(y^3)^2} = \frac{x^{2\cdot 2}}{9y^{3\cdot 2}} = \frac{x^4}{9y^6}$
$(4x^{-2}y^3)^{-3} = \frac{x^6y^9}{64}$	INCORRECT This equation is based on the *Law of Negative Exponents* as well as the *Law of Product to a Power*. The Law of Negative Exponents states that negative exponents signify division. In particular, find the reciprocal of the base ($x^{-a} = \frac{1}{x^a}$). The Law of Product to a Power states that when raising a product to a power, distribute the power to each factor ($(xy)^a = x^a y^a$). According to these laws, this equality should be:

	$(4x^{-2}y^3)^{-3} = \frac{1}{(4x^{-2}y^3)^3} = \frac{1}{4^3(x^{-2})^3(y^3)^3} = \frac{1}{64(x^{-2\cdot 3})(y^{3\cdot 3})} = \frac{1}{64x^{-6}y^9} = \frac{x^6}{64y^9}$